Number of Distant Cousins

R.E. Butler
26 June, 2011

I have been surprised by the number of contacts with distant cousins through Ancestry. I think part of the reason is that there are many more distant cousins than close cousins.
So I thought I would calculate the number of cousins of each degree, based on an average number of descendants per family.

Let c = cousin degree. In this case limited to integers. c=0 means siblings, c=1 means first cousins etc.
n = number of generations back from me. n=1 are my parents, n=2 are my grandparents.
The cousin degree is determined by the number of generations back to common ancestors.
c = n-1
Let d = number of descendants per family who have families and produce more descendants.

Siblings, c=0. n = 1, Common parents, 1 family.
If there are d descendants per family, then the number of siblings is d-1

First cousins, c = 1, n = 2 Common grandparents. 2 families of grandparents.
My father had d-1 siblings. Each of them had d children. Total number of first cousins from my father's family is d * (d-1)
My mother had d-1 siblings. Each of them had d children. Total number of first cousins from my mother's family is d * (d-1)
Total number of first cousins is 2d (d-1)

Second cousins, c= 2, n = 3 Common great grandparents, 4 families of great grandparents..
Each grandparent had d-1 siblings. Each of them had d children of my parents generation, and each of them had d children of my generation.
Total number of second cousins related to any one of my grandparents = (d-1)*d*d
There are 4 of these families, so the number of second cousins is 4 d^2 * (d-1)

General variable c. n = c+1 Common ancestors of generation n. 2^c families with children of the same generation as the common ancestors.
Each of those families produced (d-1) *d^c children of my generation who are my cousins of degree c.
The total number of cousins of degree c = (2d)^c * (d-1)

This is intuitively reasonable. If everybody has only 1 child, d = 1, and nobody has any cousins of any degree.
If the average is 2 children, then the total number of cousins of degree c is 4^c. Thus with each increase in degree, the number of cousins of that degree is 4 times as many as the previous degree.
The number of ancestors goes up by a factor of 2 for each generation further back, and the number of descendants from each goes up by a factor of d.
So if d = an average of 3 children per generation, the number of cousins of each degree is 6 times as many as the previous degree.

What is a reasonable d? Well, it currently is low, but it used to be quite a bit larger. If d=2, every 2 people produce 2 people and the population stays the same over time. Since populations actually increase, maybe a long term average might be 3. Some of my ancestors had 4 or more children who produced more children, so the average could be a lot higher.

But even with small d, the numbers increase very quickly, as shown in the following table.
If d = 2, we have 262 thousand 9th cousins.
If d = 3, we have 20 million 9th cousins….
This is another way of saying we are all related. All of us with British descent, anyway.

This is reminiscent of the problem of counting the grains of rice on a checker board. If you put 1 on the first square, 2 on the second, times 2 on each subsequent square, then ultimately the count is greater than the grain supply.
But in the case of cousins, you put d-1 siblings on the first square and multiply by 2d for every subsequent square.
Ultimately the cousin count exceeds the population….. wait a minute… there's a flaw there somewhere. ☺

Most recently, I heard from a 5th cousin. Our common ancestors had a family of 11. So I must have a huge number of 5th cousins from that family and in the end the probability of hearing from one of them is not as low as I would have previously thought.

For genealogists the message is clear…. The further back you can get, the more likely you will hear from someone…. But everybody knows that anyway…☺
And even if you have few siblings and first cousins that you know about, there are potentially a huge number you have not yet found.

Generations Back		Cousin Degree		Number of Families	d=	Number of cousins of degree c 2	3	4
1	Parents	0	siblings	1		1	2	3
2	Grandparents	1		2		4	12	24
3	Great Grandparents	2		4		16	72	192
4	…	3		8		64	432	1536
5		4		16		256	2592	12288
6		5		32		1024	15552	98304
7		6		64		4096	93312	786432
8		7		128		16384	559872	6291456
9		8		256		65536	3359232	50331648
10		9		512		262144	20155392	402653184